Bayesian structural equation models of correlation matrices: Online Supplementary Materials

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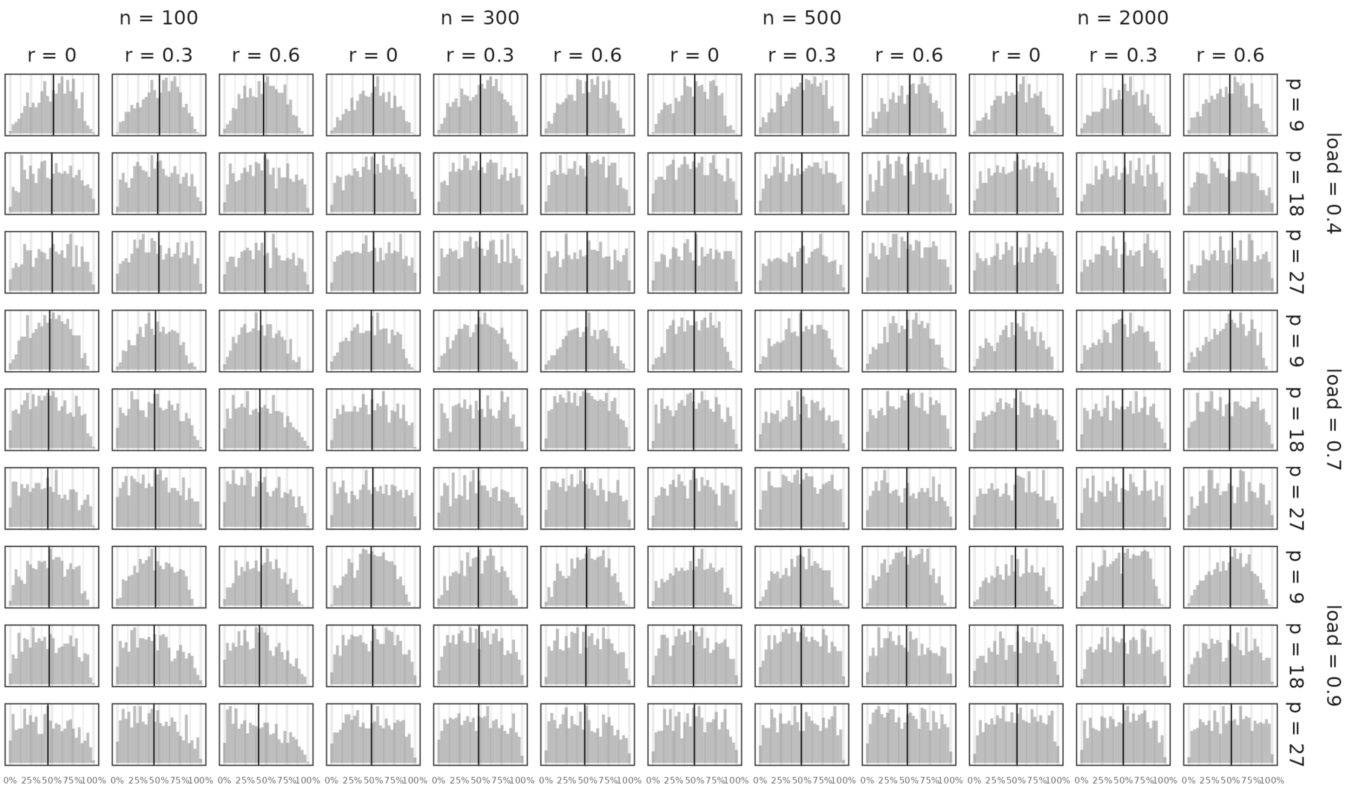
Part A: Posterior predictive p-values from simulation studies

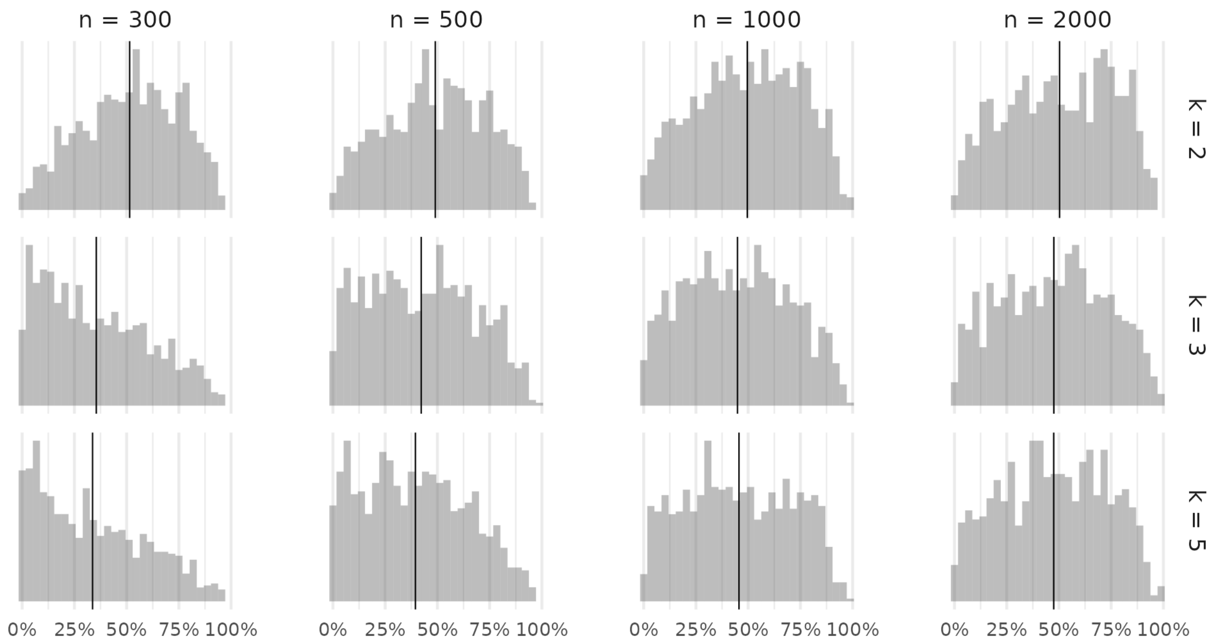
When the fitted model is correct, we can expect that pp p-values are symmetric about .5.

Figure A.1: Simulation study 1: Distribution of pp p-values



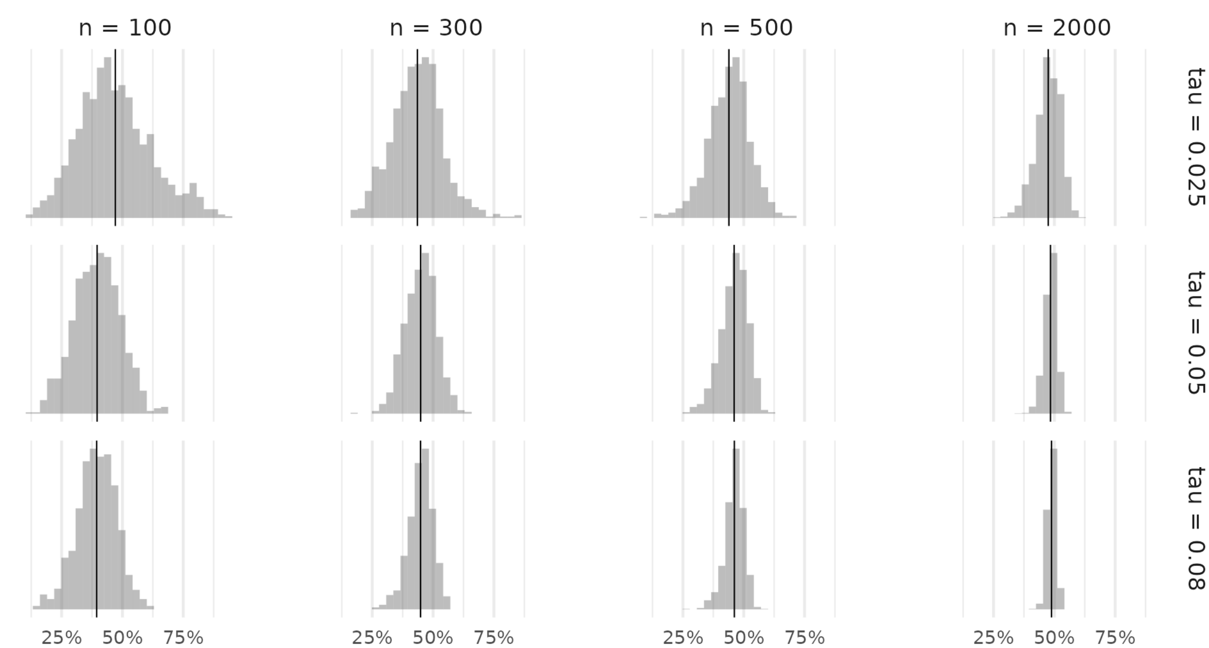
Vertical line is the mean of the pp p-values.

Figure A.2: Simulation study 2: Distribution of pp p-values



Vertical line is the mean of the pp p-values. pp p-values were inadequately right-skewed when $n = 300$ and $k \in \{3, 5\}$.

Figure A.3: Simulation study 3: Distribution of pp p-values



Vertical line is the mean of the pp p-values. Since these models account for misspecification, the pp p-values should be concentrated about .5.

Part B: RMSEA studies

B.1 Study 1 – Frequentist recovery of the RMSEA

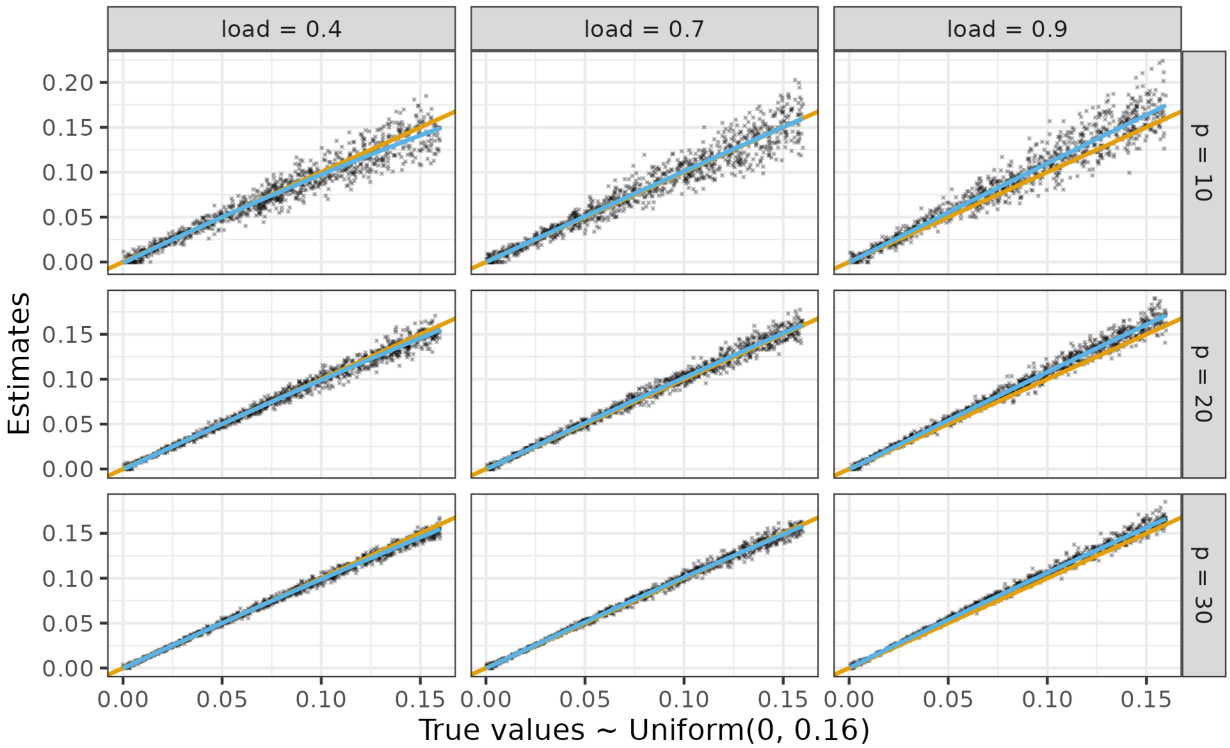
In this study, we examined an alternative data generation process (DGP) for misspecification under the log-correlation method. This DGP slightly modifies the DGP for simulation study 1 – the most important changes are highlighted below in equation 1.

$$
\begin{aligned}
\mathbf{y} &\sim \mathcal{N}_{p^*}(\boldsymbol{\gamma},\ n^{-1}\mathcal{J}\boldsymbol{\Omega}\mathcal{J}' + \boxed{\varepsilon^2\mathbf{I}_p}\ ),\ \boldsymbol{\gamma} = \gamma(\mathbf{P}(\boldsymbol{\theta})),\ \mathcal{J} = \frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{\rho}(\boldsymbol{\theta})},\ \mathbf{R} = \gamma^{-1}(\mathbf{y}),\\
&\mathbf{P}(\boldsymbol{\theta}) = \boldsymbol{\Lambda}\boldsymbol{\Phi}\boldsymbol{\Lambda}' + \boldsymbol{\Delta},\ \boldsymbol{\Lambda} = \begin{bmatrix}\boldsymbol{\lambda}_1 & \boldsymbol{\lambda}_2\end{bmatrix},\\
&\boldsymbol{\lambda}_1 = [\,\underbrace{\lambda,\ldots,\lambda}_{p/2\text{ times}},\underbrace{0,\ldots,0}_{p/2\text{ times}}\,]',\ \boldsymbol{\lambda}_2 = [\,\underbrace{0,\ldots,0}_{p/2\text{ times}},\underbrace{\lambda,\ldots,\lambda}_{p/2\text{ times}}\,]' \qquad (1)\\
&\boldsymbol{\Phi} = \left[\begin{smallmatrix}1 & \\ .3 & 1\end{smallmatrix}\right],\ \boldsymbol{\Delta} = \text{diag-matrix}\left(\text{diagonal}\left(\mathbf{I}_p - \boldsymbol{\Lambda}\boldsymbol{\Phi}\boldsymbol{\Lambda}'\right)\right)\\
&\boxed{\varepsilon \sim \text{Uniform}(0, 0.16),\ n = 5000}\ ,\ p \in \{10, 20, 30\},\ \lambda \in \{.4, .7, .9\}
\end{aligned}
$$

The variance of transformed correlations was increased by ε^2. And ε was randomly drawn from (0, 0.16) interval. Preliminary exploration suggested that ε approximately the RMSEA. Hence, we were interested in the ability of standard frequentist models to recover ε as the RMSEA. We fixed sample size at 5000 to reduce the effect of sampling variation. We varied model size and measurement quality as these parameters are known to impact the RMSEA (Savalei, 2012; Shi et al., 2019). We repeated each iteration 1000 times.

The relation between the estimated frequentist RMSEA and ε is depicted in Figure B.1. The relation between both values is well approximated by the identity function especially at lower values of ε. The approximation varied by the number of indicators and measurement quality. For example, when $\lambda = .9$, the estimated RMSEA was increasingly higher than ε at higher levels of ε; the reverse was true when $\lambda = .4$. Additionally, the noisiness of the approximation reduced as p increased, though this may simply be an effect of having more data.

Figure B.1: Recovery of ε as RMSEA from frequentist model



Orange (beneath) and blue (above) lines represent the identity function and quadratic fit respectively.

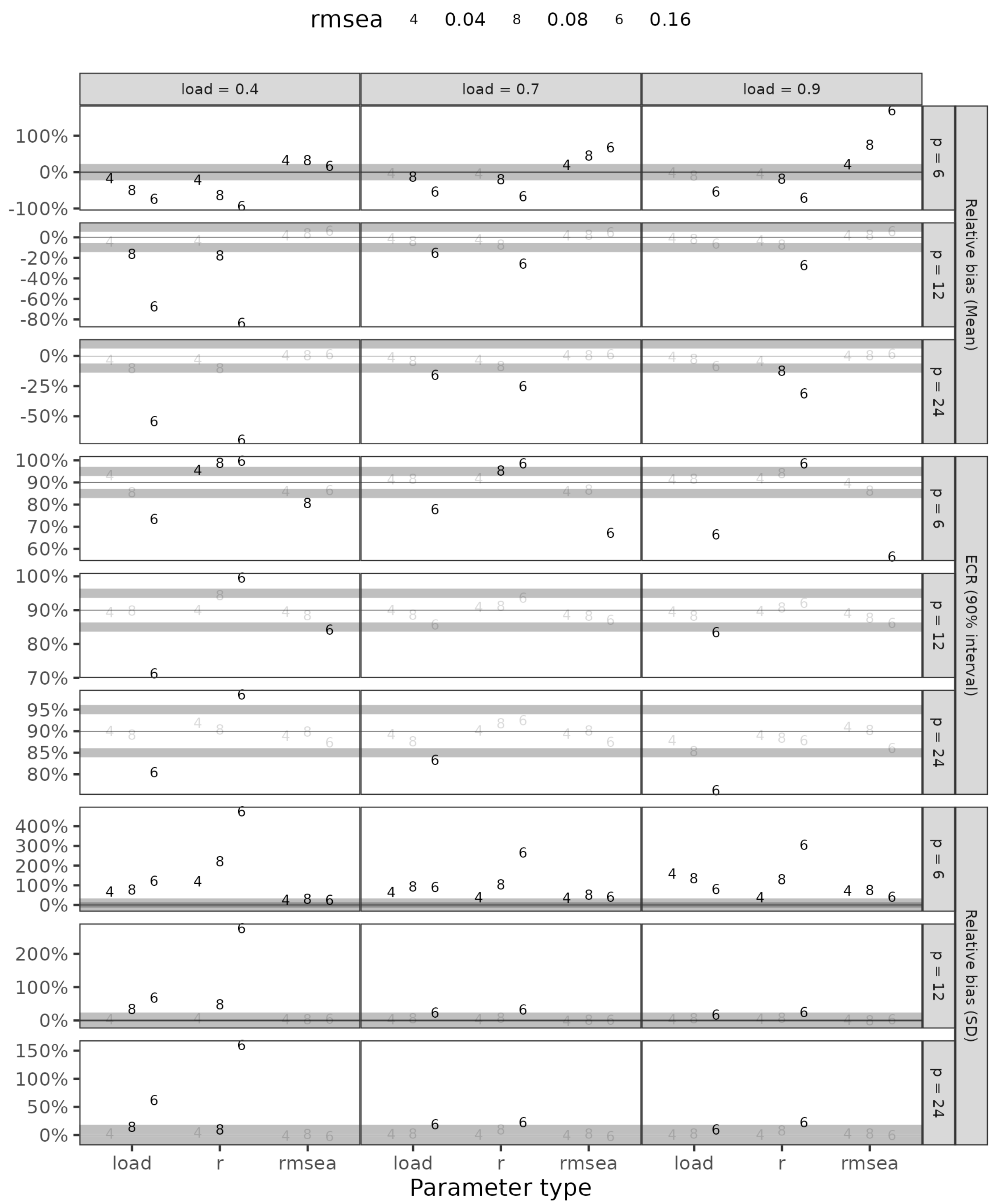
B.2 Study 2 – Modeling misspecification induced by ε

In a second study, we attempted to fit a Bayesian modeling that accounts for misspecification induced by ε. The DGP was similar to equation 1, with slight modifications: $\varepsilon \in \{0.04, .0.08, 0.16\}$, $p \in \{6, 12, 24\}$ and $n = 2000$. We reduced the number of indicators to reduce model runtimes. And the values of ε range from small to large based on conventional prescriptions. We assessed recovery of loadings on average, the interfactor correlation and ε. Results are in Figure B.2.

Parameter estimation for λ and ρ was often biased. This is unsurprising given that $\mathbb{E}(\gamma^{-1}(\boldsymbol{\gamma}+\boldsymbol{\epsilon})) \neq \gamma^{-1}(\boldsymbol{\gamma})$ even though $\mathbb{E}(\boldsymbol{\epsilon}) = 0$, as γ^{-1} is a nonlinear transformation. We can expect this inequality at larger values of ε – matching the bias patterns in Figure B.2.

On the other hand, recovery of ε was often acceptable as long as $p > 6$. In the future, it would be important to understand whether the poor recovery of ε occurs because the model has a small number of indicators, or because the nominal degrees of freedom is low.

Figure B.2: Parameter recovery when modeling misspecification induced by ε



load = average loading estimate, r = interfactor correlation. Thick lines show bounds for acceptable results as defined in the paper text. Estimates within these bounds are faded, while inadequate estimates are not to draw attention to inadequacies.

References

Savalei, V. (2012). The relationship between root mean square error of approximation and model misspecification in confirmatory factor analysis models. *Educational and Psychological Measurement*, *72*(6), 910–932. https://doi.org/10.1177/0013164412452564

Shi, D., Lee, T., & Maydeu-Olivares, A. (2019). Understanding the model size effect on SEM fit indices. *Educational and Psychological Measurement*, *79*(2), 310–334. https://doi.org/10.1177/0013164418783530