Multivariate count data analysis using Bayesian hierarchical multinomial-t compound regression: A demonstration With collocations

Bayesian Hierarchical Multinomial-t Compound

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Outline

Context for Presentation

Background Information

- Collocation is a system of words that tend to be found together, e.g. "make the bed", "do homework", ...
- Higher collocation use comes with greater language acquisition.

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- Oral interviews with 20 intermediate level speakers (L2), 20 advanced level speakers (L2) and 20 native speakers of Spanish.
- Interview duration was consistent across speakers.
- Interview text was coded for seven types of collocations.

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Statistical question

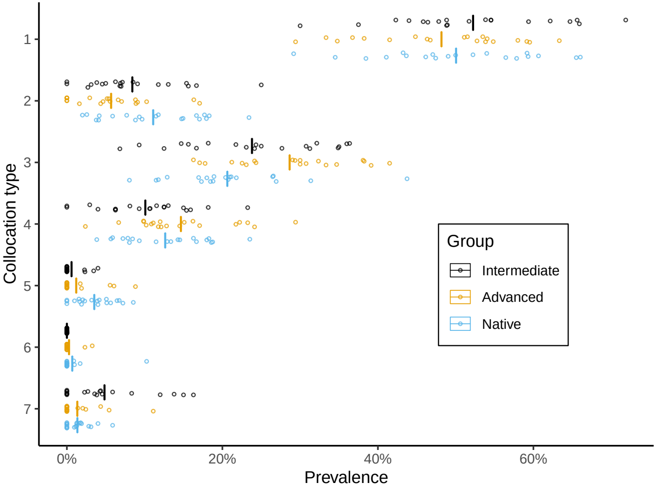
Did the three groups differ in their collocation use across the seven different collocations?

Table: Sample data

Person	Group	C1	C2	C3	C4	C5	C6	C7
i1	Intermediate	11	3	5	3	0	0	1
a1	Advanced	19	0	9	2	0	0	0
n1	Native	48	26	21	8	8	0	0

Number of times individual used collocation of a given type

Interest: Prevalence of different collocation types by group



Prevalence of collocation types (count of each collocation / total collocations) for each speaker.

Methods

Hierarchical multinomial

Rationale

- Multinomial has Poisson marginals (Townes, 2020)
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Basic model:

$$lp_{gc} = \beta_c + \delta_{gc}, \quad p_{gc} = \frac{\exp\left(lp_{gc}\right)}{\sum_{c=1}^{7} \exp\left(lp_{gc}\right)}$$

$$use_i \sim \text{Multinomial}(p_{g1}, p_{g2}, \ldots, p_{g7})$$

use_i = count vector for individual i, β_c = collocation effect (7-levels), δ_{gc} = collocation By group interaction (21-levels)

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Hierarchical priors:

$$\beta_c \sim \mathcal{N}(0, s_\beta), \quad s_\beta \sim t^+(3, 0, 1)$$

$$\delta_{gc} \sim \mathcal{N}(0, s_\delta), \quad s_\delta \sim t^+(3, 0, 1)$$

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$$use_i \sim \text{DirichletMultinomial}([p_{g1}, p_{g2}, \ldots, p_{g7}] \times \kappa_g)$$

$$\kappa_g \sim \text{Gamma}(1, 0.1)$$

κ_g = overdispersion parameter permitted to vary by group

Retained same hierarchical priors from multinomial model.
Dirichlet-multinomial (marginal likelihood) coded in Stan.

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- Poisson-normal compound to handle overdispersion (e.g. Hinde, 1982)
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Model:

$$\begin{aligned} lp_{ic} &= \beta_c + \delta_{gc} + \gamma_{ic}, \quad p_{ic} = \frac{\exp\left(lp_{ic}\right)}{\sum_{c=1}^{7}\exp\left(lp_{ic}\right)} \\ use_i &\sim \text{Multinomial}(p_{i1}, p_{i2}, \ldots, p_{i7}) \\ \gamma_{ic} &\sim t(\nu, 0, s_c),\ \nu \sim \text{Gamma}(1, 0.1),\ s_c \sim t^{+}(3, 0, 1) \end{aligned} \tag{1}$$

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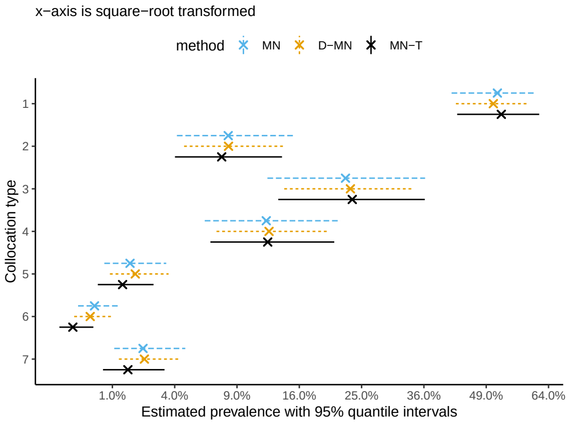
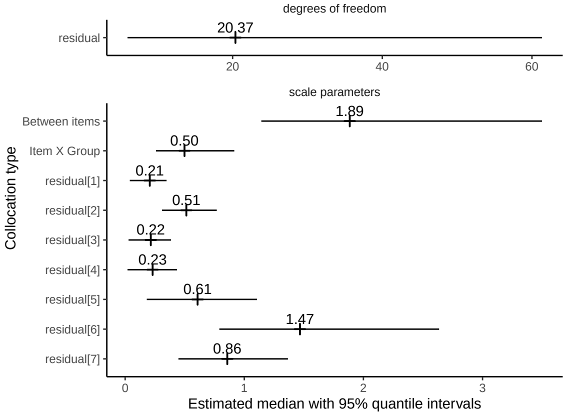
Results

Sampler: Stan (Carpenter et al., 2017), models passed both sampler-agnostic (Vehtari, Gelman, Simpson, Carpenter, & Bürkner, 2020) and sampler-specific (Betancourt, 2018) diagnostics. 1,000 post-warmup iterations across 12 chains.

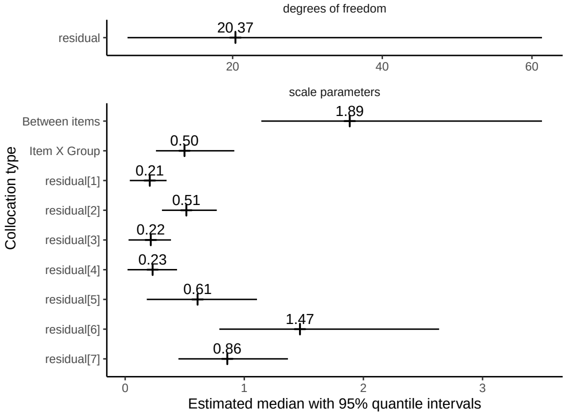


Estimated prevalence of collocation types by model. MN = multinomial, D-MN = Dirichlet-multinomial, MN-T = multinomial-t compound.

High-level model insights are about the same. Multinomial-t model has more parameters to learn from.



Degrees of freedom and scale parameters from multinomial-t model.



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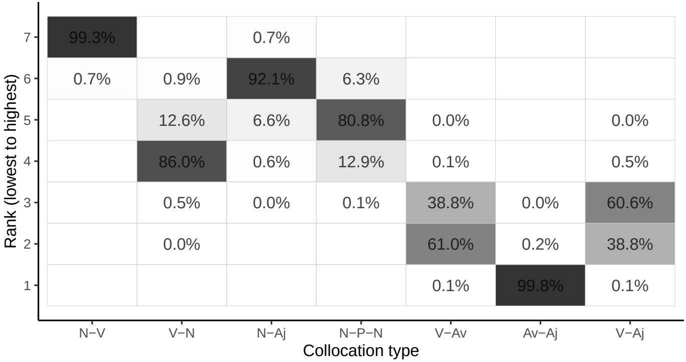
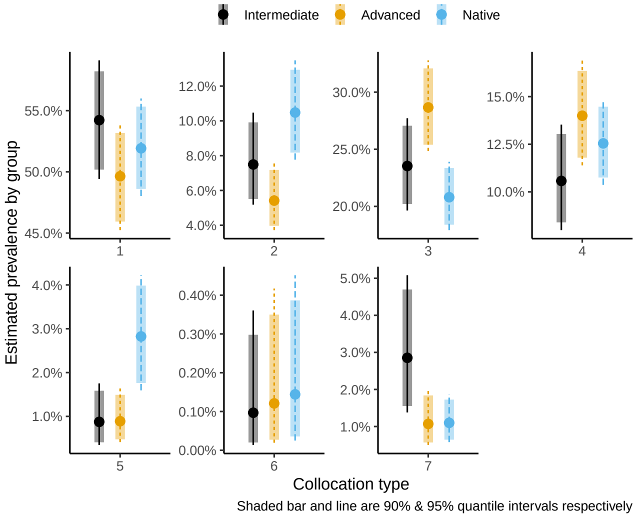
Notes

- Much of the variance is between items (collocation types), interaction accounts for less
- Residual variation differs markedly across collocation types
- Degrees of freedom is highly uncertain

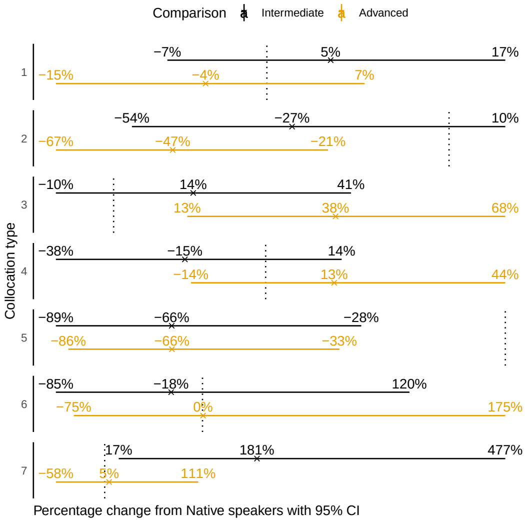
Ranking the average preference for collocations



Collocation use rate by group



Comparing L2 speakers to native speakers



In closing

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Ongoing work with the generalized Dirichlet-multinomial

- Dirichlet-multinomial imposes restrictions on the correlation between the prevalences
- Generalized Dirichlet-multinomial eases these restrictions while doubling the number of parameters - "How would hierarchical estimation proceed?"

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